HEAT AND MASS TRANSFER IN A BINARY TURBULENT BOUNDARY LAYER DURING THE FORCING OF GASES THROUGH A VERTICAL POROUS SURFACE UNDER CONDITIONS OF NATURAL CONVECTION

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UDC 532.517.2:536.25

An approximate analytical solution of heat and mass transfer is given for a binary turbulent boundary layer developing during natural convection at a vertical surface.

An attempt is made directly to calculate the parameters of permeability as a function of the tangential stress and heat flux according to an equation obtained for forced flow.

An analysis of the solution is presented with the aim of determining the contribution of the various physical parameters to heat transfer during the forcing of helium and air.

The solution obtained is compared with a solution achieved earlier by one of the authors, where the permeability factor was calculated indirectly through the thickness of the boundary layer and the characteristic velocity.

Data are presented on an experimental study on the forcing of CO_2 and He through a vertical porous surface under conditions of natural convection. The experimental results are in good agreement with the analytical solution obtained.

A determination is made of the limits at which there develops an inversion effect associated with a change in the direction of the velocity in the viscous sublayer of the turbulent boundary layer during the forcing of CO_2 through a heated vertical surface.

EFFECT OF RADIATION ON HEAT TRANSFER IN TUBES OF CIRCULAR CROSS SECTION

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The steady flow of a viscous liquid with constant physical properties in a circular channel, in the presence of radiative heat exchange, is examined in the article. If the liquid is diathermal the effect of heat transfer by radiation on the convective heat exchange appears through the boundary conditions [1].

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It is assumed that the radiative heat transfer takes place only in the plane of each cross section, i.e., the heat generated in the channel walls in a given section is given off by radiation only in that section. The error resulting from such an assumption will be smaller, the smaller the relative width of the channel and the variation in wall temperature along the length of the channel in comparison to its mean value.

In such a formulation the problem under examination reduces to an integral energy equation [1]

$$AW_xR = \frac{\partial T}{\partial R}\left[R\left(1+\frac{a_T}{a}\right)\right]\frac{\partial T}{\partial R} \tag{1}$$

with following boundary conditions:

$$\left(R\frac{\partial T}{\partial R}\right)_{R=R_1} = -\frac{q_1 r_1}{\lambda} + \frac{\sigma_0 \varepsilon_{re} r_1}{\lambda}(T_1^4 - T_2^4),$$
$$\left(R\frac{\partial T}{\partial R}\right)_{R=1} = \frac{q_2 r_2}{\lambda} - \frac{\sigma_0 \varepsilon_{re} r_2}{\lambda}(T_2^4 - T_1^4), \tag{2}$$
$$(a_T)_{R=R_1,\,1} = 0, \quad \frac{1}{\varepsilon_{re}} = \frac{1}{\varepsilon_2} + \frac{F_1}{F_2}\left(\frac{1}{\varepsilon_2} - 1\right).$$

Having integrated Eq. (1) twice within the limits from $R_1 = r_1/r_2$ to R, taking into account the boundary conditions (2), we obtain

$$T - T_1 = \int_{R_1}^{R} \frac{A\int_{R_1}^{R} W_x R\,dR}{\left(1+\frac{a_T}{a}\right)R} + \int_{R_1}^{R} \frac{\left[-\frac{q_1 r_1}{\lambda} + \frac{\sigma \varepsilon_{re} r_1}{\lambda}(T_1^4 - T_2^4)\right]}{\left(1+\frac{a_T}{a}\right)R}\,dR. \tag{3}$$

If the difference in wall temperatures $T_1 - T_2$ is small in comparison with T_1 and T_2 then as an approximation we can take

$$T_1^4 - T_2^4 \approx 4T_W^2(T_1 - T_2),$$

where $T_W = (1/2)(T_1 + T_2)$, and we write Eq. (3) in the form

$$\theta = \frac{2}{1-R_1^2}\int_{R_1}^{R} \frac{\int_{R_1}^{R} W_x R\,dR}{\left(1+\frac{\nu_T}{\nu}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\right)R}\,dR + \int_{R_1}^{R} \frac{\left[4\Phi\theta_{12} - \frac{1}{1+\beta}\right]}{\left(1+\frac{\nu_T}{\nu}\frac{\mathrm{Pr}}{\mathrm{Pr}_T}\right)R}\,dR, \tag{4}$$

where

$$\theta = \frac{(T - T_1)\lambda}{q_1 r_1 + q_2 r_2}.$$

Thus, the determination of the temperature field comes down to a calculation of the integrals in Eq. (4). Moreover, we can obtain integral equations for the heat-exchange coefficients at the inner and outer walls of the channel α_1 and α_2. In the absence of radiant transfer ($\Phi = 0$) these equations convert into the well-known equations for convective heat exchange in tubes of circular cross section [2].

The laminar movement of a diathermal gas in a circular channel is examined as a specific example.

NOTATION

r	is the flow radius of the tube;
r_1	is the inner radius of the tube;
r_2	is the outer radius of the tube;
$d_{eq} = 2(r_2 - r_1)$	is the equivalent diameter;
$R = r/r_2$	is the dimensionless radius of the tube;
$R_1 = r_1/r_2$	is a geometrical parameter;
l	is the channel length;
w_x	is the velocity of liquid;
$\overline{w}$	is the average velocity of the liquid;
$W_x = w_x/\overline{w}$	is the dimensionless velocity of the liquid;
T	is the liquid temperature;
T_1, T_2	are the temperatures of the inner and outer walls;

$\bar{T}$	is the calorimetric average of the liquid temperature;
q_1, q_2	are the heat flux densities at the inner and outer walls;
$q_{21} = q_2/q_1$	is the ratio of heat fluxes;
F_1, F_2	are the areas of the inner and outer walls;
$\varepsilon_1, \varepsilon_2$	are the emittance of the inner and outer walls;
ε_{re}	is the reduced emittance;
$\sigma_0 = 4.9 \cdot 10^{-8}$ kcal/m^2 · h · deg	is the Stefan–Boltzmann constant;
$\alpha_1 = q_1/(T_1 - T), \alpha_2 = q_2/(T_2 - T)$	are heat-exchange coefficients at the inner and outer walls;
λ	is the heat-conduction coefficient;
ν_T	is the coefficient of turbulent transfer of the amount of movement;
α_T	is the coefficient of turbulent heat transfer;
$Pr = \nu/\alpha$	is the Prandtl number;
$Pr_T = \nu_T/\alpha_T$	is the turbulent Prandtl number.

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DESIGN OF THERMAL INSULATING SHIELDS FOR CRYOGENIC VACUUM CHAMBERS

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A zonal method based on the calculation of the averaged characteristics of radiant heat transfer is used for a calculation of the basic parameters of chevron shields for cryogenic vacuum chambers: the transmission capacity α and the transmission coefficient of thermal radiation η.

The plane problem of radiant heat transfer to an element of an optically opaque chevron shield (Fig. 1) consisting of the edge surfaces F_b, F_c, F_d, and F_e, the entrance surface F_a, and the exhaust surface F_h is examined with a derivation of the equations. For a more precise calculation the surfaces F_a and F_h are divided into m identical sections, designated as F_k and F_i, while the surfaces F_b, F_c, F_d, and F_e are divided into n sections, designated as F_l, F_p, F_q, and F_r.

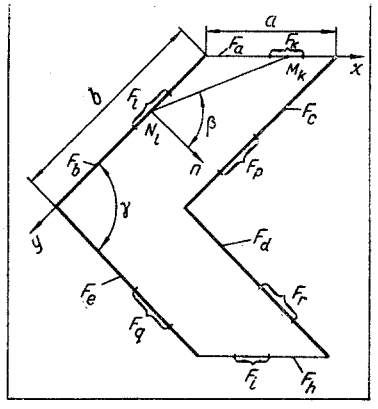

Fig. 1

The calculating equations obtained have the form

$$\eta = \eta_1 + \eta_2, \tag{1}$$

$$\eta_1 = \frac{1}{m} \sum_{k=1}^{m} \sum_{i=1}^{m} \Phi_{ki}, \tag{2}$$

As a result of Eq. (2)

$$\eta_2 = \frac{1}{m} \frac{E_{\text{int. sh}}}{E_{\text{res. a}}} \sum_{i=1}^{m} \sum_{l,\,p,\,q,\,r=1}^{n} \sum_{j=l,\,p,\,q,\,r} \Phi_{ij}. \tag{3}$$

Here Φ_{ki} is the mean allowed angular coefficient of radiation from the surface F_k to the surface F_i; Φ_{ij} is the mean allowed angular coefficient of radiation to the surface F_j (j = l, p, q, r) from the surface

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F_i; $E_{res.a}$ is the resultant radiation flux density of the surface F_a; $E_{int.sh}$ is the internal radiation flux density of the shield.

The transmission capacity α is also determined from Eq. (2); in this case it is assumed that the exhaust gases do not condense on the shield surfaces (the coefficient of reflection of its surfaces is R = 1), while the conditions of interaction of the molecular fluxes from the surfaces of the shield are identical to the conditions for radiant heat transfer.

Calculating equations are presented for the angular coefficients of radiant heat transfer between the surface elements of the shield which are necessary for computing the allowed angular coefficients Φ_{ki} and Φ_{ij}. Calculations were made on a computer to determine the transmission capacity α and the coefficient of transmission of thermal radiation η_1 as a function of the coefficient of reflection of the shield R and its geometrical dimensions (ratio of edge length to spacing b/*a* and angle between edges γ). The results of the calculation are presented in the form of graphs of $\eta_1(b/a, \gamma, R)$ and $\alpha(b/a, \gamma)$.

TEMPERATURE FIELD OF MEASURING COMPARTMENT IN HEATED FILAMENT METHOD

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UDC 536.2.08

In an experimental determination of the coefficient of thermal conductivity of gases and liquids by the heated filament method it is assumed that the outer resistance thermometer measures the temperature of the outer surface of the glass capillary.

In reality the thermal resistance will be lower in the zones of contact of the thermometer coils with the capillary than where it is not in contact. Therefore, the temperature field will not be uniform along the length of the measuring compartment and the temperature measured by the resistance thermometer will differ from the average temperature at the outer surface of the capillary.

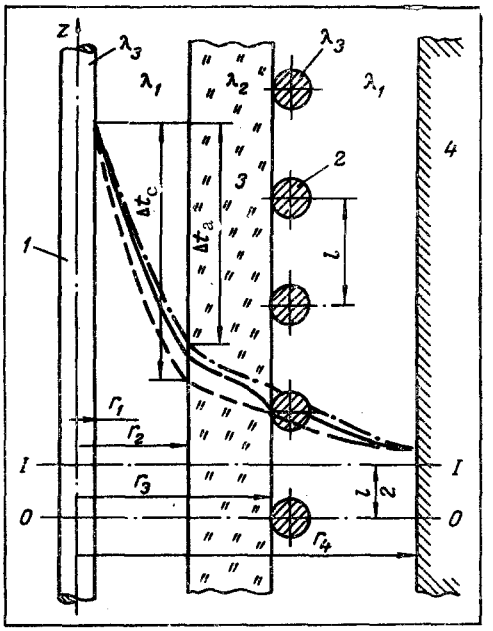

Fig. 1. Diagram of measuring compartment: 1) platinum heating filament; 2) platinum filament of outer resistance thermometer; 3) glass capillary; 4) thermostatic block. Solid line: temperature in section 0–0; dash and dot line: average temperature; dashed line: computed temperature.

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For a quantitative estimate of the effect of nonuniformity of the temperature field on the accuracy of determining the coefficient of thermal conductivity the problem of the temperature distribution in the measuring compartment in the heated filament method was solved on a computer by the grid method.

In solving the problem the following assumptions were made: 1) the outer thermometer is wound on the capillary not in the form of a spiral but in the form of rings with a spacing l; 2) the temperature of the inner resistance thermometer filament is constant and equal to t_1 (Fig. 1); 3) the temperature of the autoclave wall is also constant and equal to t_2; 4) the temperature field is symmetrical with respect to the axis of the measuring compartment; 5) the heat in the layer between the measuring compartment and the autoclave is transferred by thermal conduction; 6) the thermal conduction coefficients of the medium being studied λ_1, the glass capillary λ_2, and the platinum λ_3 do not depend on the temperature; 7) there is no heat flux through the sections 0–0 and I–I (Fig. 1), i.e., $-\lambda(\partial t/\partial z)|_{z=0} = 0$ and $-\lambda(\partial t/\partial z)|_{z=l/2} = 0$.

The plane problem of thermal conduction obtained taking into account the different thermal conduction coefficients and the curvature was solved by the grid method with steps of 0.02 mm.

The calculations showed that, as a result of distortion of the temperature field, the platinum filament of the outer resistance thermometer has a lower temperature than the average temperature of the outer surface of the capillary. The nonuniformity of the temperature field decreases at the capillary wall, which leads to a more uniform temperature distribution in the measuring space. Graphs of the temperature variation along the radius of the measuring compartment, and the actual and computed temperature drops Δt_a and Δt_c are shown in Fig. 1. Since $\Delta t_a < \Delta t_c$, the experimental values of the thermal-conduction coefficient obtained are too low. The error in determining the thermal-conduction coefficient depends on the dimensions of the measuring compartment, the spacing of the outer resistance thermometer winding, and the thermal-conduction coefficient of the medium being studied. The error grows with an increase in the thermal-conduction coefficient of the substance studied and decreases with an increase in the diameter of the capillary.

For a measuring compartment with dimensions $r_1 = 0.05$ mm, $r_2 = 0.47$ mm, $r_3 = 0.85$ mm, $r_4 = 6.21$ mm, and $l = 1.36$ mm the error in a calculation of the thermal-conduction coefficient without taking into account the nonuniformity of its temperature field is 0.8% for air, 1.3% for toluene, and 1.9% for water.

DETERMINATION OF HEAT RELEASE FUNCTION IN COMBUSTION CHAMBERS WITH TRANSIENT FUEL SUPPLY

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UDC 536.14

From the point of view of energetics a heat release function can be represented in the form of a delayed function of the fuel supply which is the potential carrier of the energy of heat release:

$$\xi = \sigma(\tau - \tau_i). \tag{1}$$

During the period τ_i the breakup of the fuel stream into individual drops, the formation and development of the fuel plume, the heating and vaporizing of the individual drops, the diffusion of the vapors and formation of the burning mixture, and finally the burning process itself take place. It should be noted that not all the heat introduced with the fuel is used in the combustion chamber. Part of the heat is lost because of heat exchange and from blowout through combustion chamber leakage. These losses can be expressed through a supplemental period $\Delta\tau_i$ of the delay argument, causing an equivalent decrease in the heat release function. The nature of the function $\tau_i = f(\tau)$ can be expressed through the initial parameters and

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boundary conditions. The boundary conditions are determined by the parameters of the fuel-supply system, the physicochemical parameters of the fuel, and the temperature of the surrounding walls. The initial conditions are determined by the gas–thermodynamic parameters of the air and the geometrical parameters of the fuel plumes at the moment of initial combustion.

The following generalized equation of relative heat release is obtained for diesel combustion chambers:

$$\xi = \xi_l - m \sin 2\pi\xi_l, \tag{2}$$

where

$$\xi_l = \frac{\Pi_a}{\varphi_{in}}(\varphi_{i0} - \varphi_\tau) + \frac{\Pi_c}{\varphi_{in}}[1 - \exp \Pi_d (\varphi_{i0}^{\Pi_n} - \varphi_\tau^{\Pi_n})] \tag{3}$$

represents the relative heat release for a linear fuel supply function.

For existing diesels the combustion chamber criteria which can be determined depend on the construction parameters of the chamber and the fuel apparatus and on the gas–thermodynamic parameters of the air charge.

The criterion Π_a characterizes the heat loss due to heat exchange and blowout through piston ring leakage, Π_c how fully the fuel injected into the combustion chamber is used which depends on the condition of the sprayer, Π_d the initial reaction rate, depending on the fuel, the volume of the plumes, and the duration of the injection (the heat release rate grows with an increase in Π_d), and Π_n the turbulent diffusiontime of the air charge into the burning zone (the heat release rate decreases with increase in Π_n). Π_n decreases with an increase in the coefficient of air surplus and the rate of movement of the air charge.

For existing undivided diesel combustion chambers Π_a = 0.008–0.028, Π_c = (0.9–1.0) φ_{in}, Π_d = 8–28, Π_n = 1.1–2.3, and m = 0–0.1.

NOTATION

τ	is the running time from the moment of fuel injection;
τ_{i0}	is the delay period of self-ignition (induction period);
τ_i	is the delay period of heat release;
σ	is the time function of the relative fuel supply;
φ_τ, φ_{i0}, φ_i	are the relative values of the parameters τ, τ_{i0}, τ_i reduced to the period of the working stroke of the diesel;
φ_{in}	is the reduced fuel injection time;
Π_a, Π_c, Π_d, Π_n	are empirical dimensionless parameters characterizing the combustion chamber;
m	is a characteristic of the fuel-supply system.

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INDUCTION ACCELERATION OF CONDUCTORS AND OF PLASMA

P. M. Kolesnikov, N. S. Kolesnikova, and I. B. Gavris

UDC 538.323;533.9

The paper presents a study of transient electromagnetic processes and the acceleration of a plasma in a pulsed accelerator with inductive coupling of the plasma and the primary circuit.

As a mathematical model we consider the transient processes in the primary electrical circuit, consisting of a stationary coil, in which a capacitor bank is discharged through a resistance and an inductance. A current is induced in a movable coil, which is coaxial with the stationary coil, during the current discharge; its action with the currents in the primary circuit leads to the acceleration of the movable coil. The fundamental equations that describe the induction acceleration of a single coil and the transient processes in the accelerator circuit, in dimensionless form, are

$$\frac{d^2y}{d\tau^2} = q\frac{|\varphi_1'|\,|\varphi_2'|\,y}{\sqrt{1+y^2}}\left[-K+\frac{a_1+y^2}{a_2+y^2}E\right], \tag{1}$$

$$\frac{d\varphi_1'}{d\tau}+a_3\frac{d}{d\tau}\left\{\varphi_2'\left[\left(\frac{2}{k}-k\right)K-\frac{2}{k}E\right]\right\}+\alpha_1\varphi_1'-\varphi_1=0, \tag{2}$$

$$\frac{d\varphi_1}{d\tau}=-\varphi_1', \tag{3}$$

$$\frac{d\varphi_2'}{d\tau}+a_4\frac{d}{d\tau}\left\{\varphi_1'\left[\left(\frac{2}{k}-k\right)K-\frac{2}{k}E\right]\right\}+\alpha_2\varphi_2'=0, \tag{4}$$

where y is the path traversed by the plasma being accelerated; y' is the velocity of the plasma; φ_1' is the current in the primary circuit; φ_2' is the current in the movable circuit; φ_1 is the voltage across the windings of the capacitor; K and E are elliptic integrals of the first kind and of the second kind, respectively; k is their modulus; and τ is the time. The quantities q, a_1, a_2, a_3, a_4, a_5, α_1, α_2 are dimensionless parameters.

The system of equations (1)-(4) was solved numerically for repeatedly varying values of the parameters with the initial conditions: for $\tau = 0$, we have $\varphi_1' = \varphi_2' = y' = 0$; $\varphi_1 = 1$; $y = 0.1$.

As a result of the investigation of the system of equations (1)-(4) we establish that, in induction accelerators, the plasma velocities attain maximum values in a shorter time, the value of the parameter q has a weaker effect on the relative variation of the velocity compared with the usual electrode accelerator, and the variation in velocity rapidly attains its maximum value (Fig. 1). In Fig. 1 the numerals on the curves correspond to the values of the parameters: $a_1 = a_3 = a_4 = 0.5$; $\alpha_1 = \alpha_2 = 0.1$; 1) q = 1, a_2 = 0; 2) q = 4, a_2 = 0; 3) q = 1, a_2 = 0; 4) q = 1, a_2 = 1; 5) q = 4, a_2 = 1; 6) q = 10, a_2 = 1.0.

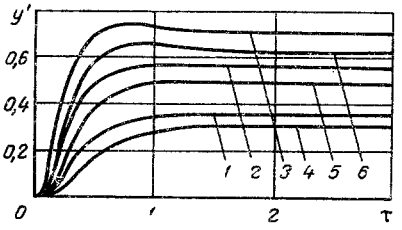

Fig. 1

On the curve of the variation of the discharge current in the primary circuit near the current maximum a dip appears. This is characteristic for many experimental devices; the dip indicates that there is an induction interaction in the accelerators. The discharge currents in the movable circuit are comparable in magnitude with the discharge currents in the primary circuit.

It is shown that using such a scheme we can describe and model the acceleration stratified on separate plasmoids, interacting with each other according to an induction method, and an estimate is made of the effect of the motion of one plasmoid on the others, and on the entire system.

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ACCELERATION OF A PLASMA IN A COAXIAL ACCELERATOR WITH AN OPTIMAL INDUCTIVE ENERGY ACCUMULATOR

P. M. Kolesnikov, N. S. Kolesnikova, and I. B. Gavris

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The paper considers the combined operation of a coaxial plasma accelerator and an induction energy accumulator with a ferromagnetic core for minimum energy losses in the induction accumulator, when the current source ensures the following law of current variation [1]:

$$I = A(\mathrm{sh}\, at + b\, \mathrm{ch}\, at), \tag{1}$$

where I is the current of the charge cycle; A, a, and b are constants; and t is the time. In this case the equations describing the transient processes in the circuits and the acceleration of the plasma with account of mass-transport processes, in dimensionless variables, take the form

$$\frac{dy\mu}{d\tau} = q\varphi'^2, \tag{2}$$

$$\frac{dy}{d\tau} = y', \tag{3}$$

$$\frac{d\mu}{d\tau} = \pm\gamma_1\mu - \gamma_2\mu^2 + \gamma_3\varphi'^2 + \gamma_4|\varphi'| - \gamma_5\mu^3 + \gamma_6 y', \tag{4}$$

$$(1+y)\frac{d\varphi'}{d\tau} + y'\varphi' + p\varphi' - \beta\,\mathrm{ch}\,\tau - \alpha\,\mathrm{sh}\,\tau = 0, \tag{5}$$

where y is the path traversed by the plasma of mass μ to be accelerated; y' is the velocity of the plasma; τ is the time; φ' is the discharge current; and q, $\alpha, \beta, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6$ are dimensionless parameters of the system. Equations (2)-(3) are the law of motion of the plasma under the action of magnetic-pressure forces; Equation (4) describes the kinetics of the mass-transfer processes in the plasma, taking account of diffusion of particles, two-particle and three-particle recombination of plasma, anode and cathode sputtering, and the drag of the neutral gas [2]; Equation (5) is the second Kirchhoff law for a charge–discharge circuit.

The system (2)-(5) was solved numerically for the variable quantities that appear in its parameters and the initial conditions: for $\tau = 0$ we have $y = y' = \varphi' = 0$, $\mu = 1$.

Results of the numerical investigation show that an induction energy accumulator having an optimal law of current variation in the charging circuit both substantially changes the character of the transient processes in the accumulator–accelerator system, and also improves the characteristics of the plasma being accelerated; it can ensure an increase in the velocity of the plasma of several orders of magnitude in comparison with the same accelerator, but having a nonoptimal law of variation in the charging circuit [3]. In the figures curves are presented for the variation of y, y', and φ' as functions of the parameters of the system and of the time. A calculation of the transient processes in an accelerator with an induction accumulator is presented, and the characteristics of the accelerators being investigated are compared. The effect of mass-transport processes on the characteristics of the plasma being accelerated is investigated; it is shown that these processes have a considerable effect.

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THE THEORY OF HEAT TRANSFER IN NUCLEATE POOL BOILING

A. A. Voloshko

UDC 536.423.1

An analytical method for calculating the heat flux density from a heating surface is described on the basis of a theoretical and experimental study of the physical characteristics of nucleate boiling. The method is based on the equation

$$q = \frac{\pi}{6} f d_0^3 n_F (r\rho'' + c_m \rho' c' \delta T), \tag{1}$$

in which the first term represents the heat flux density associated with phase conversion and the second term corresponds to the heat flux density spent in heating the liquid transported by vapor bubbles from the wall boundary layer into the main volume.

In the calculation of the heat-transfer rate a relation proposed by Labuntsov [1] is used to estimate the density of active vaporization centers, and an expression derived in [2] is used to determine the average volumetric velocity of the vapor phase released from one center. In dimensionless form the equation appears as follows:

$$\mathrm{Nu}_* = \frac{20\pi}{3} C_0 C_1 \sqrt{2\varphi(\theta)}\, \mathrm{Kt}^2 \mathrm{Ja}^2 \left(1 + c_m \frac{\delta T}{\Delta T} \mathrm{Ja}\right). \tag{2}$$

From Eq. (2) we obtain

$$\text{for } c_m \frac{\delta T}{\Delta T} \mathrm{Ja} \ll 1 \quad \mathrm{Nu}_* = \frac{20\pi}{3} C_0 C_1 \sqrt{2\varphi(\theta)}\, \mathrm{Kt}^2 \mathrm{Ja}^2, \tag{3}$$

$$\text{for } c_m \frac{\delta T}{\Delta T} \mathrm{Ja} \gg 1 \quad \mathrm{Nu}_* = \frac{20\pi}{3} C_0 C_1 c_m \frac{\delta T}{\Delta T} \sqrt{2\varphi(\theta)}\, \mathrm{Kt}^2 \mathrm{Ja}^3. \tag{4}$$

Equations (3) and (4) show that the power exponent of the Jakob number Ja changes in general, depending on the values of Ja, which in turn depends significantly on the saturation pressure. The criterion Ja is a measure of the ratio between the quantity of heat spent in heating unit volume of the liquid and the volumetric heat of vaporization. Equations (3) and (4) therefore reflect the dominant influence of either of the two indicated terms of the total heat flux density.

We have compared the criterial equation (2) with the results of voluminous experimental studies for a wide range of the controlling parameters. We obtained satisfactory qualitative and quantitative agreement.

NOTATION

$\mathrm{Nu}_* \equiv \frac{\alpha}{\lambda'} \sqrt{\frac{\sigma}{(\rho' - \rho'')g}}$;

$\mathrm{Kt} \equiv \frac{(r\rho'')^2}{\rho' c' \sigma T_s} \sqrt{\frac{\sigma}{(\rho' - \rho'')g}}$;

$\mathrm{Ja} \equiv (\rho' c' \Delta T)/\rho'' r$;

α is the heat-transfer coefficient;

q is the heat flux density;

λ' is the thermal conductivity of the liquid;

ρ', ρ'' are the densities of the liquid and vapor;

σ is the coefficient of surface tension;

r is the latent heat of vaporization;

c' is the specific heat of the liquid;

$\Delta T = T_w - T_s$;

T_w is the wall temperature of the heating surface;

T_s is the saturation temperature;

$\delta T = T_f - T_s$;

T_f is the temperature of the liquid in the wall layer;

Technical Institute of Fishing and Fisheries, Astrakhan'. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 21, No. 6, p. 1118, December, 1971. Original article submitted December 29, 1970; abstract submitted March 4, 1971.

g	is the acceleration of gravity;
d_0	is the bubble breakoff diameter;
n_F	is the density of active vaporization centers;
f	is the bubble breakoff frequency;
θ	is the extremal macroscopic wetting angle;
$\varphi(\theta)$	is a function of the limiting angle;
C_0, C_1, c_m	are numerical coefficients.

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INCEPTION OF FORCED-CIRCULATION NUCLEATE BOILING OF A LIQUID

V. A. Chernobai

UDC 536.423.1

An analytical solution is given for the problem of determining the appropriate conditions for the inception of forced-circulation nucleate boiling.

A solution scheme is investigated for determining the bubble apex temperature and the temperature gradient as a function of the apex coordinate; the scheme is analogous to one proposed by M. V. Aleksandrov. In the present study, however, another equation is used for the temperature field and the temperature gradient in the liquid, and a different technique is used to account for convective heat transfer and its relationship with the hydrodynamics of forced circulation. These modifications make it possible to deduce equations that can readily be solved for any of the variables characterizing the conditions for the inception of nucleate boiling: q_{ib}, $T_w^{ib}-T_s$, ΔT_{uh}^{ib}, and ρW_{ib}.

The final solution is tested by processing of the author's and other researchers' experimental data for water, ethylene glycol, ethyl alcohol, and butyl alcohol. The deviation of the experimental data does not exceed ±15%.

The results of a comparison of the solution for q_{ib} and $T_w^{ib}-T_s$ with the calculated relations recommended by other authors show that the relations proposed in the article well describe the conditions for the inception of nucleate boiling of a liquid in the case of forced circulation.

NOTATION

q	is the heat flux density;
T	is the temperature;
ρ	is the density of the liquid;
W	is the velocity of the liquid.

Subscripts and Superscripts

ib	denotes inception of boiling;
w	denotes the wall;
s	denotes saturation;
uh	denotes underheating.

Kiev Polytechnic Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol.21, No.6, p.1119, December, 1971. Original article submitted December 17, 1970; abstract submitted April 5, 1971.

TEMPERATURE FIELDS AND STRESSES IN JOINED PLATES HAVING DIFFERENT HEAT-EXCHANGE COEFFICIENTS

Yu. M. Kolyano and L. A. Gavur*

UDC 539.377

A system of two isotropic semiinfinite plates of uniform material butt-joined using a thin interstitial layer having internal thermal resistance is examined in the work. Heat exchange takes place through the surface $z = \pm\delta$ of the system with the external medium according to Newton's law. The temperatures of the media flowing over each plate and the coefficients of heat exchange with the lateral surfaces of the plates are different and constant. The initial temperature t_{in} is assumed to be constant. A solution of the boundary problem for the system under examination is obtained using a Laplace transformation with respect to time. The temperature field is presented for the case when the temperature of the external medium is the same everywhere. In particular, when the thermal resistance of the interstitial layer is equal to zero the temperature field takes the form

$$T_i = t_e + (t_{in} - t_e)\exp(-a\varkappa_i^2\tau)\operatorname{erf}\left(\frac{x}{2\sqrt{a\tau}}\right) + \frac{1}{\pi}(t_{in} - t_e)$$

$$\times \exp(-a\varkappa_i^2\tau)\int_0^\tau \exp\left[a(\varkappa_i^2 - \varkappa_{i\pm1}^2)\zeta - \frac{x^2}{4a(\tau-\zeta)}\right]\frac{d\zeta}{\sqrt{\zeta(\tau-\zeta)}},$$

where τ is the time; t_e is the temperature of the external medium; $\varkappa_i = \sqrt{\alpha_i/\lambda\delta}$, α_i, λ, and a are coefficients of heat exchange, thermal conduction, and thermal diffusion; and 2δ is the thickness of the plates.

Numerical calculations of the temperature field and temperature stresses at the junction boundary of the plates are presented for the latter case. In this connection it is assumed that the Biot number Bi_1 is equal to unity at the surface of the lower plate, while for the surface of the upper plate the Biot numbers are taken as $Bi_2 = 0$, 0.1, 0.5, and 1.

NONSTATIONARY HEATING OF A LAMINAR STRUCTURE WITH A CURRENT

I. A. Zhvaniya and G. A. Tkhor†

UDC 536.2

The question of nonstationary heating of nonuniform media has been widely discussed in the literature [1, 2], but little attention has been paid to current systems. As is known, the Peltier effect occurs when a current flows through the contact between two materials in the presence of a material with a thermal electromotive force different from zero, while Joule and Thompson heat is evolved in the volume of the materials (we shall restrict the examination to isotropic bodies).

The boundary problem of nonstationary thermal conduction for a laminar structure with a current is solved first in the article. The solution is found by a method developed by A. Datsev [3]. A system of integral equations of Volterra's second kind which always has a solution is obtained for the contact temperatures of the neighboring layers. Knowing these solutions, the spatial–temporal temperature distribution is easy to find in the form of quadratures. The solution of the stationary problem is found as a particular case (asymptotic).

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COOLING AND HEATING OF FLAT STEEL BARS

N. I. Yalovoi, L. V. Sudoplatov, and N. Yu. Taits

UDC 669.18-41.065

The temperature field of a steel bar before it is put into the heating oven depends on a multitude of prior factors, principal amongst which are the duration of standing in the mold and of cooling in the air of the bars.

In the article an attempt is made to obtain a general analytical solution incorporating all the thermal processes taking place after the flat bar fully solidifies – cooling in the mold and in the air and heating under the roller – since in practice these processes are organically related to one another. In this case we assume that the bar is fully solidified while in the mold (consequently, the more complicated case of the air cooling and oven heating of a bar with a liquid core is excluded from the examination). We also assume that:

a) the bar consists of an unbounded plate;

b) the temperature distribution through a section of the bar conforms to a linear function at the moment of its full solidification (this is taken as the starting moment for our problem);

c) heat exchange between the surface of the bar and the medium surrounding it takes place according to Newton's law, where the heat-exchange coefficient and the temperature of the surrounding medium are known functions of the time;

d) the thermophysical properties of the bar material are constant.

A function of the bar temperature, which was found using an integral Laplace–Carson transformation, is written in the following form:

$$v(X, \mathrm{Fo}) = \int_0^{\mathrm{Fo}} \mathrm{Bi}(t)\,[v_c(t) - v_{X=1}]\,\Phi_1(\mathrm{Fo} - t,\ X)\,dt - \Delta v_0\,[\Phi_2(\mathrm{Fo},\ X) - \Phi_3(\mathrm{Fo},\ X)] + \Delta v_0\,(1 - X), \tag{1}$$

where

$$\Phi_1(\mathrm{Fo},\ X) = 1 + 2\sum_{k=1}^{\infty} (-1)^k \exp(-\pi^2 k^2\,\mathrm{Fo}) \cos \pi k X;$$

$$\Phi_2(\mathrm{Fo},\ X) = \mathrm{Fo} + \frac{1}{3} - X + \frac{X^2}{2} - 2\sum_{k=1}^{\infty} \frac{\exp(-\pi^2 k^2 \mathrm{Fo})}{\pi^2 k^2} \cos \pi k X,$$

$$\Phi_3(\mathrm{Fo},\ X) = \mathrm{Fo} - \frac{1}{6} + \frac{X^2}{2} - 2\sum_{k=1}^{\infty} (-1)^k \frac{\exp(-\pi^2 k^2\,\mathrm{Fo})}{\pi^2 k^2} \cos \pi k X,$$

v(X, Fo) is the dimensionless temperature function; X is a relative coordinate; Bi(Fo) is the Biot number; Fo is the Fourier number; v_m(Fo) is the dimensionless temperature of the surrounding medium; Δv_0 is the initial temperature drop across the section of the bar.

M. I. Arsenichev Dneprodzerzhinsk Industrial Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 21, No. 6, pp. 1120-1121, December, 1971. Original article submitted May 7, 1970; abstract submitted February 22, 1971.

The unknown function $v_{X=1}$ which enters into Eq. (1) must be determined from an integral Volterra equation of the second kind, which can be obtained if we set X = 1 in Eq. (1).

NONSTEADY TEMPERATURE FIELD OF A BODY WHEN THE THERMAL-CONDUCTIVITY COEFFICIENT OF THE MATERIALS IS A FUNCTION OF TEMPERATURE

O. T. Il'chenko and L. I. Shifan

UDC 536.21

The problem considered is that of the temperature field in a solid body for boundary conditions that vary with time, when the thermal-conductivity coefficient of the material is a function of the temperature of the body, and the variation of the thermal-diffusivity coefficient is such that we can assume that it is equal to the average value over the entire range of variation.

Applying a Kirchhoff transformation to the original system of equations in the case of a linear temperature dependence for the thermal-conductivity coefficient, we obtain

$$\frac{\partial\bar{\Phi}(\bar{x},\ \tau)}{\partial\tau}=\bar{a}_{av}^2\frac{\partial^2\bar{\Phi}(\bar{x},\ \tau)}{\partial\bar{x}^2}\ , \tag{1}$$

$$\bar{\Phi}(\bar{x},\ 0)=0, \tag{2}$$

$$\frac{\partial\bar{\Phi}(0,\ \tau)}{\partial\bar{x}}=0, \tag{3}$$

$$\frac{\partial\bar{\Phi}(1,\ \tau)}{\partial\bar{x}}=\mathrm{Bi}(\tau)\left\{\sqrt{\frac{1}{b(\Phi_s-\Phi_n)}\left[\frac{1}{b(\Phi_s-\Phi_n)}+\frac{2\Phi_n}{(\Phi_s-\Phi_n)}+2\right]}\right.$$

$$\left.-\sqrt{\frac{1}{b(\Phi_s-\Phi_n)}\left[\frac{1}{b(\Phi_s-\Phi_n)}+\frac{2\Phi_n}{(\Phi_s-\Phi_n)}+2\bar{\Phi}(1,\ \tau)\right]}\right\}, \tag{4}$$

where

$$\Phi(\bar{x},\ \tau)=T(\bar{x},\ \tau)+\frac{bT^2(\bar{x},\ \tau)}{2}\ ;\quad a_{av}^2\cong\frac{\lambda(T)}{C(T)\gamma(T)}\ ;$$

$$\bar{\Phi}(\bar{x},\ \tau)=\frac{\Phi(\bar{x},\ \tau)-\Phi_n}{\Phi_s(\tau)-\Phi_n}\ .$$

We assume a solution of Eq. (1) with boundary conditions (2)-(4) by the method discussed in [1, 2] by means of successive approximations of the nonlinear boundary condition (4).

As a zeroth approximation we assume a linear boundary condition, when $\mathrm{Bi}_{cond}(\tau)=\mathrm{Bi}(\tau)$. In problems with $T_c(\tau)=T_c^{max}=\mathrm{const}$, the zeroth approximation of condition (4) has the form

$$\frac{\partial\bar{\Phi}(1,\ \tau)}{\partial\bar{x}}=\mathrm{Bi}(\tau)[1-\bar{\Phi}(1,\ \tau)]. \tag{5}$$

The method discussed is illustrated by examples of the solution of problems with boundary conditions that are both constant and variable with time. The results of the calculation are compared with the data of electromodeling on a grid according to the method of Libman.

Polytechnical Institute, Khar'kov. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 21, No. 6, pp. 1121-1122, December, 1971. Original article submitted August 4, 1970; abstract submitted March 31, 1971.

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PROGRESSION OF THE CRYSTALLIZATION FRONT DURING CONVECTIVE COOLING

Yu. S. Postol'nik

UDC 536.248.2

The problem of the progression of the crystallization front is solved approximately by the method of averaging functional corrections.

The paper derives an expression which defines the position of the crystallization front $l(\tau)$ during solidification of a plate (m = 0), a cylinder (m = 1), or a sphere (m = 2):

$$H^*(l)\,[1+\beta_1(l)+\beta_2(l)]=\frac{\tau}{K_a\,\mathrm{Ko}}\,, \tag{1}$$

where

$$H^*(l)=\frac{2+\mathrm{Bi}_1}{2(1+m)\,\mathrm{Bi}_1}\left[1-\frac{2(1-m+\mathrm{Bi}_1)(1-l)^{1+m}-(1+m)\,\mathrm{Bi}_1(1-l)}{(1-m)(2+\mathrm{Bi}_1)}\right]; \tag{2}$$

$$\beta_1(l)=\frac{2(2+\mathrm{Bi}_1)[2m+(1+m)\,\mathrm{Bi}_1]}{3\mathrm{Ko}\mathrm{Bi}_1^3\,H^*(l)}\left\{\left[l-\frac{2}{\mathrm{Bi}_1}\ln\left(1+\frac{\mathrm{Bi}_1 l}{2}\right)\right]\right.$$

$$\left.+\frac{6[(1+m)\,\mathrm{Bi}_1^2-(1+2m)\,\mathrm{Bi}_1-2m]\,\mathrm{Bi}_1 l^2-4[(1+2m)\,\mathrm{Bi}_1-m]\,\mathrm{Bi}_1^2 l^3+3m\,\mathrm{Bi}_1^3 l^4}{12(2+\mathrm{Bi}_1)[2m+(1+m)\,\mathrm{Bi}_1]}\right\}; \tag{3}$$

$$\beta_2(l)=\frac{8[(1-m)(3+\mathrm{Bi}_1)-3(1-m+\mathrm{Bi}_1)(1-l)^{2+m}+(2+m)\,\mathrm{Bi}_1(1-l)^3]}{9(2+m)\,\mathrm{Po}\,[(1-m)(2+\mathrm{Bi}_1)-2(1-m+\mathrm{Bi}_1)(1-l)^{1+m}+(1+m)\,\mathrm{Bi}_1(1-l)^2]}; \tag{4}$$

$$K_a=\frac{a_2}{a_1}\,;\quad \tau=\frac{a_2 t}{R^2}=\mathrm{Fo};\quad \mathrm{Ko}=\frac{q_c\gamma_2}{\gamma_1 c_1(T_c-T_m)}\,;$$

$$\mathrm{Po}=\frac{q_c}{c_2(T_0-T_c)}\,;\quad l(\tau)=\frac{r(t)}{R}\,; \tag{5}$$

q_c is the latent heat of crystallization; a_ν is the thermal diffusivity; γ_ν is the density; c_ν is the specific heat; T_0, T_c, T_m are the initial, the crystallization, and the ambient temperature, respectively; r(t) is the thickness of the solidified layer; 2R is the plate thickness or the cylinder (sphere) diameter; the subscript $\nu = 1$ refers to the solid (frozen) phase and subscript $\nu = 2$ refers to the liquid (wet) phase.

The additive terms $\beta_1(l)$ (3) and $\beta_2(l)$ (4) appearing in Eq. (1) account for the actual heat in the solid phase and the initial superheat in the liquid phase, respectively.

For $l(\tau_c = 1)$, from Eqs. (1) and (2) we get the following formula determining the time τ_c of complete crystallization:

$$\tau_c=\frac{2+\mathrm{Bi}_1}{2(1+m)\,\mathrm{Bi}_1}K_a\,\mathrm{Ko}\,[1+\beta_1(1)+\beta_2(1)]. \tag{6}$$

The structure of Eqs. (1) and (6) confirms the validity of the recommended procedure for taking into account the heat contents of the liquid and the solid phase by means of corresponding increases in the latent heat of crystallization.

M. I. Arsenichev Dneprodzerzhinsk Industrial Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 21, No. 6, pp. 1122-1123, December, 1971. Original article submitted December 8, 1970; abstract submitted March 10, 1971.

For practical use, all relations are represented in the form of graphs which facilitate calculations with any values of parameters K_a, Ko, Po, and Bi_1. The procedure for graphical solution is illustrated on a specific numerical example.

It is noted that disregarding the heat contents of the solid and the liquid phase may result in large errors (46% in the illustrative example).

DIFFUSION IN BINARY SYSTEMS IN A NONUNIFORM MAGNETIC FIELD

L. S. Atroshchenko, S. M. Voronina, and V. N. Panasenko

UDC 538.4

The diffusion process is investigated in a binary mixture the components of which have different magnetic susceptibilities. The magnetic susceptibility of one component is greater than that of the other. Diffusion takes place in a closed volume in a nonuniform magnetic field, bringing about a redistribution of the components in the mixture.

We consider the plane problem, where the nonuniformity of the magnetic field is directed along the x axis. Chemical reactions do not take place between the components, and there are no sources or sinks in the volume. The diffusivity and magnetic susceptibility are constant.

It is assumed that at the initial time the components are uniformly distributed throughout the entire volume. Upon application of the field, which is a maximum at the coordinate origin, the components become redistributed in the volume with the passage of time.

When dynamic equilibrium is reached, the concentration attains a constant value in the steady-state concentration profile at the concentration limits.

The problem is investigated for the case in which the magnetic field strength is described by the function

$$H(x) = H_0 \sqrt{1 - \frac{x}{l}},$$

In the solution of the problem the eigenvalues and eigenfunctions are determined. A table of eigenvalues is given.

The solution of the problem consists of two terms, one of which describes the concentration profile in the interior volume when dynamic equilibrium is reached, while the other describes the time variation of the concentration field.

A numerical calculation is carried out for a binary mixture having definite physical properties. The concentration profile of the mixture components as a function of the magnetic field strength is obtained. The domain of applicability of the resulting formulas is determined. A procedure is given for the approximate calculation of the concentration profile outside the domain of applicability of the formulas.

Donets State University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 21, No. 6, pp. 1123-1124, December, 1971. Original article submitted July 3, 1970; abstract submitted April 29, 1971.

DISPERSE COMPOSITION OF DROPLETS FORMED IN THE DISINTEGRATION OF A JET

V. B. Lemberskii and M. B. Ferber

UDC 532.522

The size distributions of droplets are determined on the basis of the analogy between a system of droplets and a gas or are described by empirical equations. Neither approach takes the jet disintegration process into account. On the other hand, observations of the disintegration of jets show that the jet becomes detached at locations of wave separation. Therefore, the size distribution of the droplets is characterized by wave growth on the surface of the jet. Inasmuch as the amplitude variation is determined by the growth rate α, it is necessary to associate with the probability density function W for the formation of a droplet of diameter δ the corresponding coefficient α for the given diameter.

In the elementary case of axisymmetric perturbations the dependence of the density function on the coefficient α has the form

$$W = \begin{cases} \alpha^2(\delta)/N & \text{for } \delta \geqslant \delta_0, \\ 0 & \text{for } \delta < \delta_0, \end{cases} \tag{1}$$

where $\delta_0 = 1.68d$ is the minimum droplet diameter, and $N = \int_0^{\delta_0} \alpha(\delta)d\delta$ is a normalization factor.

Substituting the formula for $\alpha^2(\delta)$ [1] into Eq. (1), we obtain

$$W = \begin{cases} 122\frac{d^5}{\delta^6}\left[1-22{,}2\left(\frac{d}{\delta}\right)^6\right] & \text{for } \delta \geqslant \delta_0, \\ 0 & \text{for } \delta < \delta_0, \end{cases} \tag{2}$$

where d is the diameter of the jet.

In the case of atomization the density function W is represented as follows:

$$W = \begin{cases} \alpha(\delta)/N_1 & \text{for } \delta_{\min} \leqslant \delta \leqslant \delta_{\max}, \\ 0 & \text{for } \delta < \delta_{\min},\ \delta > \delta_{\max}, \end{cases} \tag{3}$$

where $N_1 = \int_{\delta_{\min}}^{\delta_{\max}} \alpha(\delta)d\delta$ is a normalization factor; $\delta_{\min} = (B/\rho We)^{1/3}$; $\delta_{\max} = (B/3)^{1/3}d$ are the minimum and maximum droplet diameters; and B is an empirical coefficient.

Describing the dependence $\alpha(\delta)$ by an expression derived in [2], we find

$$W = \begin{cases} \frac{B(d/\delta)^3}{N_1}\left(A - \frac{B(d/\delta)^3}{Re}\sqrt{We}\right) & \text{for } \delta_{\min} \leqslant \delta \leqslant \delta_{\max}, \\ 0 & \text{for } \delta < \delta_{\min},\ \delta > \delta_{\max}. \end{cases} \tag{4}$$

where $A = \sqrt{(We/Re^2)B^2(d/\delta)^6 - B(d/\delta)^3 + \rho We}$; $We = (\rho_1 u^2 d)/2\sigma$ is the Weber number; $Re = \rho_1 ud/2\mu_1$ is the

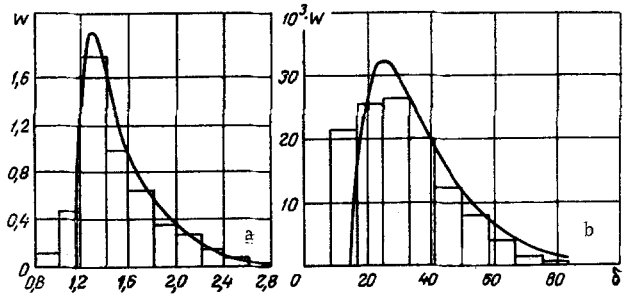

Fig. 1. Probability density function W versus diameter δ of the drops formed. a) Histogram based on authors' experimental data; δ, mm; solid curve calculated according to Eq. (2); b) based on data of [2]; δ, μ; solid curve calculated according to Eq. (4); W, mm^{-1}.

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Reynolds number; $\rho = \rho_2/\rho_1$; ρ_2 is the density of the liquid; ρ_2 is the density of the surrounding medium; μ is the liquid exit flow velocity; σ is the coefficient of surface tension; and μ_1 is the dynamic viscosity coefficient.

The calculations according to Eq. (2) are compared with the authors' own experimental data, and those according to Eq. (4) are compared with the data of [2] on the atomization of diesel fuel (Fig. 1). The analytical and experimental results exhibit good agreement.

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GRAVITY DISCHARGE OF A LOOSE MATERIAL FROM A CLOSED CONTAINER

V. E. Davidson, A. P. Tolstopyat, and N. P. Fedorin

UDC 532.529.5

Experiments were performed using a flat bin with transparent walls and a slot through the bottom. The slot width was $b \leq 0.83f$ and the slot length varied within $0.15 \leq b/a \leq 4.0$; f denotes the bin width and a, b the respective slot dimensions. Quartz sand was used as the loose material. The flow rate of the loose material was determined by recording its dropping level in the bin cinematographically and also by the weighing method. A comparison between $K(\delta)$ from an open bin (1) and a closed bin (2), where K denotes the dimensionless flow rate (per sec) of the loose material and δ denotes the reduced diameter of the outlet hole in the bin bottom [1], is shown in Fig. 1. The lower dimensionless flow rate from the closed bin is explained by a rarefaction above the surface of loose material due to the evacuation of the bin. The accompanying pressure drop generates a countercurrent of air, which in turn reduces the flow rate of the loose material. The vertical distribution of air pressure across the layer of loose material during discharge from the closed bin is shown in Fig. 2. As can be seen here, at $\bar{H} < 0.5$ ($\bar{H} = h/b$ is the referred height of the layer) the gage pressure $\bar{p}$ is zero, i.e., atmospheric pressure prevails in the region below the dynamic concave surface of loose material and, consequently, the hypothesis proposed in [2] that there exists a pressure gradient in the space below the concave surface has not been confirmed by our experiment. The effect of a discharge hole on the pressure distribution in a layer of loose material extends to the height $\bar{H} \approx 2$.

Since the flow rate of loose material is lower from a closed bin than from an open bin, we tried to recover the drop in the flow rate by injecting air through the lid of the bin. The flow rate of air necessary

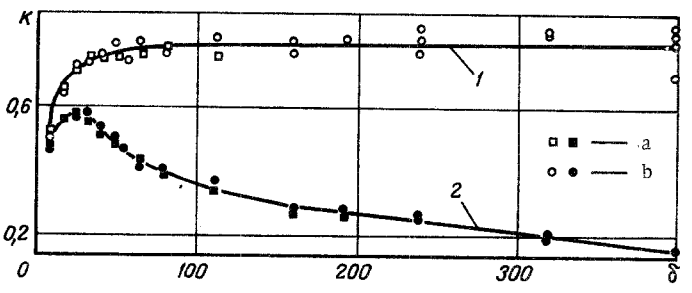

Fig. 1. Comparison between dimensionless flow rates of loose material from an open and a closed bin: 1) open bin; 2) closed bin; a) weighing method; b) cinematography.

Three-Hundred-Years-of-Union-between-the-Ukraine-and-Russia Dnepropetrovsk State University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 21, No. 6, pp. 1125–1126, December, 1971. Original article submitted November 11, 1970; abstract submitted February 22, 1971.

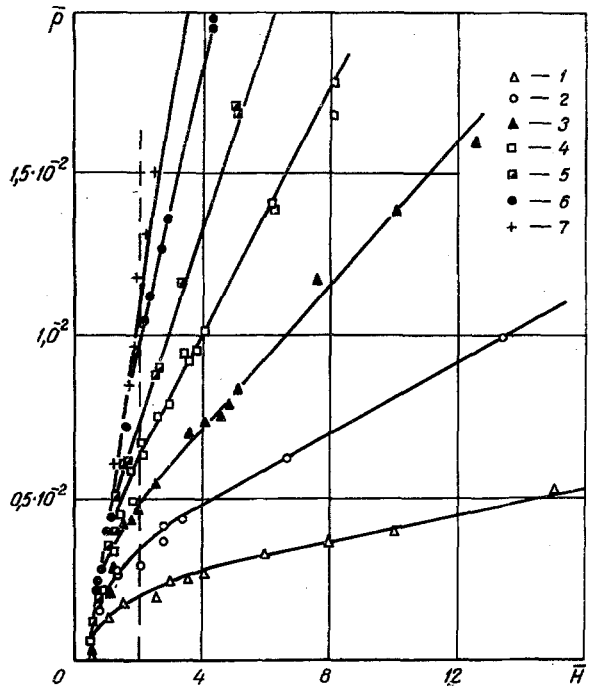

Fig. 2. Pressure distribution in a layer of loose material: 1) δ = 24; 2) 40; 3) 64; 4) 79; 5) 95; 6) 111; 7) 127. The dashed line indicates the boundary of the region affected by the presence of the discharge hole.

for restoring the flow rate of loose material is related to the latter as follows:

$$G_g = 8\cdot10^{-4}G,$$

where G_g is the flow rate of gas and G is the mass flow rate of loose material.

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THE PHENOMENON OF HYDRAULIC INDUCTION IN THE TRANSIENT FLOW OF AN INCOMPRESSIBLE VISCOUS FLUID

G. G. Zel'kin

UDC 532.5

The method of transfer characteristics has been proposed in [1] and developed in [2, 3] for studies of the transient flow of incompressible viscous fluids through hydro systems with long pipelines and local constrictions, but it is evident that the results of experiments cannot be adequately explained in the light of generally known physical concepts.

As the studies have shown, during a change in the flow rate of a liquid there forms (is induced) a transient vortex Φ. The energy spent on producing an induced vortex during any kind of change in the flow rate impedes this change, i.e., $\Delta p_{L_V} = -d\Phi/dt$, where Δp_{L_V} denotes the loss of pressure head in the hydro

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system on inducing the vortex Φ. The vortex and the flow rate of a liquid are related by a simple proportion: $\Phi = L_v Q$. Coefficient L_v will be called the vortical inductance.

When the flow of a liquid is transient, the inertia of the liquid behaves in an analogous manner as the induced vortex. The energy spent on overcoming the inertia of a liquid during any kind of change in the flow rate impedes this change, i.e., this energy is reactive in nature – like the energy spent on inducing a vortex. The impedances to flow due to inertia, therefore, as well as the impedances due to vortex induction, are reactive hydraulic impedances, and we will call them hydraulic inductive reactances [4]; they may be characterized by the hydraulic inductance L_h, which is made up of the vortical inductance L_v and the inertial inductance L_i.

As a unit of hydraulic inductance we have chosen the value at which a 1 kg/cm^2 ($\sim 10^5$ N/m^2) pressure head is lost when the flow rate of a liquid under transient conditions changes uniformly by 1 m^3/sec per second. This unit has been called the king (koéffitsient induktsii gidraulicheskoi – hydraulic induction coefficient).

In addition to the hydraulic reactances, the flow of an incompressible viscous fluid also encounters impedances due to friction forces and eddies, and the energy spent on overcoming those is converted to heat. These impedances have been called hydraulic resistances.

The ratio of hydraulic reactances to hydraulic resistances determines the length of the transient period. This ratio is expressed mathematically as $t = L_h/R_h$, where the quantity R_h defines the hydraulic resistances.

The phenomenon of hydraulic induction which we have discovered leads us now to a new equation which will fully describe the processes occurring during transients in the flow of an incompressible viscous fluid through hydro systems with long pipelines and local constrictions. Unlike the well-known fundamental equation for the transient flow of an incompressible viscous fluid, the new equation contains an additional term which accounts for the energy lost in inducing a vortex.

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